Multiple Dielectric Posts in a Rectangular Waveguide

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Abstract —This paper presents a complete analysis for a system of linear, homogeneous, and isotropic dielectric posts in a rectangular waveguide. These posts are assumed uniform along the narrow side of the waveguide, but are otherwise of arbitrary cross section and thickness. The scattering and impedance matrices describing the effect of the posts on the dominant waveguide mode are derived. The latter is then realized as a two-port T-network. A moment procedure is devised and applied to a set of test problems with a wide variety of post configurations to compute the scattering parameters and equivalent network elements. The accuracy and convergence aspects of the numerical solution are also investigated. The branch and network resonances are determined for some post configurations.

I. Introduction

CONSIDER a system of dielectric posts $P^1, P^2, \cdots, P^p$ in a rectangular waveguide. These posts are assumed uniform along the narrow side of the waveguide, but are otherwise of arbitrary cross section and thickness. The waveguide medium is assumed linear, homogeneous, isotropic, and dissipation free, and is therefore characterized by the real scalar constitutive parameters (μ, ϵ). The dielectric posts are likewise linear, homogeneous, and isotropic, although not necessarily free from losses, and are therefore characterized by the constitutive parameters (μ, ϵ^m), where ϵ^m, $m = 1, 2, \ldots, p$, are complex scalars. The problem considered is shown in Fig. 1. The solution involves determining the equivalent network describing the effect of the posts on the dominant waveguide mode.

Over the past few years, the study of inductive metallic posts in a rectangular waveguide has been an area of active research. A solution for a system of inductive posts of arbitrary cross section and thickness was given by Auda and Harrington [1]. Leviatan, Li *et al.* [2], [3] reported solutions for single and multiple posts of circular cross section. Less attention, however, has been paid to the study of dielectirc posts, despite its theoretical and practical significance. In essence, metallic posts are the limiting case of the dielectric ones as the imaginary parts of the dielectric constants tend to $-\infty$. Furthermore, the dielectric posts, unlike the metallic ones, are very resonant structures. The study of the resonance phenomenon is therefore an important part in the analysis of dielectric posts. On the practical side, ceramic dielectrics with high dielectric constant and temperature stability have now become available commercially. This has made it possible to replace many bulky and expensive waveguide resonant cavities in the design of microwave filters and highly stable microwave oscillators by low-cost miniature dielectric resonators [4]. Dielectric posts can be used in such applications as well, in particular, in situations where high power handling capability is required.

The literature on dielectric posts in a rectangular waveguide is scarce, with almost all existing solutions dealing exclusively with the problem of a single centered post of circular cross section. Schwinger was the first to solve the single centered circular post problem using his celebrated variational method [5, ch. 2]. Although ingenious and powerful, the application of the method was limited to posts of small radii and relatively low permittivities. The few data collected can be found in Marcuvitz's *Waveguide Handbook* [6, sec. 5-12]. Araneta *et al.* [7] recently attempted to improve on Schwinger's results by incorporating one more trial term in the variational expressions for the network parameters. They also provided conditions for the permittivity for which the branches of the equivalent network become resonant. Modal field expansion and matching of boundary conditions were utilized by Ikegami [8] and Cicconi and Rosatelli [9] to solve for a centered circular post of plasma with a parabolic dielectric susceptibility profile. Nielsen [10] used the modal expansion method to treat the problem of a centered circular homogeneous plasma post covered with a dielectric. It has recently come to our knowledge that Leviatan and Scheaffer are currently considering solutions for systems of dielectric posts [11]. Their approach is a generalization of the method in [2], and appears to have the same merits of convergence and accuracy.

This paper presents a complete analysis for the system of dielectric posts. A volume integral field equation for the equivalent polarization current for each post is derived, and is later solved numerically using a subsectional point-matching moment procedure. The scattering and impedance matrices describing the effect of the posts on the dominant waveguide mode are obtained in terms of the equivalent currents. The latter is then realized in the form of a two-port T-network using standard microwave network theory. The moment procedure is used to compute the scattering parameters and equivalent network elements for a wide variety of post configurations, thereby providing a large set of design data that have not been available

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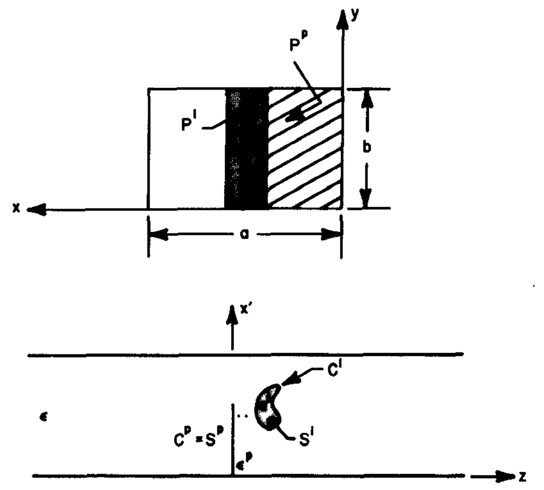


Fig. 1. $[p]$ uniform dielectric posts in a rectangular waveguide.

before. Resonances are then determined from the data obtained.

The organization of the paper is as follows. A complete field analysis leading to the integral equation for the equivalent currents is presented in the next section. The scattering matrix of the posts is obtained in Section III followed by the derivation of the equivalent network in Section IV. The numerical solution of the integral equation and evalution of matrix elements are considered, respectively, in Sections V and VI, and some of the results obtained are given in Section VII. In Section VIII, the resonances are defined and determined for some post configurations.

II. Basic Formulation

Let a dominant waveguide mode of unit amplitude be incident on the posts from the left. This mode has only a y-component of electric field given by [12, sec. 4-3]

$$E_y^i = \sin\left(\frac{\pi}{a}x\right)e^{-\gamma_1 z} \tag{1}$$

where

$$\left.\begin{aligned}\gamma_1 &= j\frac{2\pi}{\lambda_1} = j\sqrt{\kappa^2 - \left(\frac{\pi}{a}\right)^2}\\ \kappa &= \frac{2\pi}{\lambda} = \omega\sqrt{\mu\epsilon}\end{aligned}\right\}. \tag{2}$$

In (2), κ is the wave number of the waveguide medium, and λ is its vavelength. Furthermore, it is assumed that $a < \lambda < 2a$, so that only the dominant mode can propagate in the waveguide.

Since each post is uniform along the y-axis, and since the exciting mode has only a y-component of electric field that does not vary with y, so does the scattered field. The only higher-order modes excited are therefore TE_{n0} to z modes since these are the only waveguide modes having only a y-component of electric field that does not vary with y. Furthermore, the electric polarization vector associated with the mth post P^m has only a y-component given by [13, sec. 1-6]

$$P_y^m = \epsilon_0(\epsilon_r^m - \epsilon_r)E_y^m = -j\omega\mu\epsilon_0(\epsilon_r^m - \epsilon_r)\phi^m \tag{3}$$

where ϵ_0 is the permittivity of a vacuum, ϵ_r^m is the relative permittivity or dielectric constant, and E_y^m and ϕ^m are the y-components, respectively, of the electric field and magnetic vector potential inside the post. The effect of P^m on the incident mode can then be completely accounted for by an equivalent distribution of electric polarization current of density

$$J_y^m = j\omega P_y^m = \left(\frac{\omega}{c_0}\right)^2(\epsilon_r^m - \epsilon_r)\phi^m \tag{4}$$

where $c_0 = 2.997925\times10^8$ m/s is the velocity of light in a vacuum.

Let the dominant mode be incident in the waveguide, while the posts are replaced by $\bigcup_{m=1}^{P} J_y^m$. (As can be seen, the problem is basically a two-dimensional scalar one that can be entirely worked out in some constant y-plane within the waveguide. Henceforth, all source and field points are assumed located in such a plane.) The field scattered in the waveguide is then identical with the field produced by the polarization currents and can therefore be determined in terms of these currents using the Green's function for TE_{n0} to z modes in a rectangular waveguide. Thus,

$$E_y^s = -j\omega\mu\left(\frac{\omega}{c_0}\right)^2\sum_{l=1}^{p}(\epsilon_r^l - \epsilon_r)\cdot\int_{S^l}\phi^l(x',z')G(x,z|x',z')\,ds' \tag{5}$$

where

$$\left.\begin{aligned}G(x,z|x',z') &= \frac{1}{a}\sum_{n=1}^{\infty}\frac{1}{\gamma_n}\sin\left(n\frac{\pi}{a}x\right)\cdot\sin\left(n\frac{\pi}{a}x'\right)e^{-\gamma_n|z-z'|}\\ \gamma_n &= \sqrt{\left(n\frac{\pi}{a}\right)^2 - \kappa^2}\end{aligned}\right\} \tag{6}$$

is the Green's function for the TE_{n0} to z modes [14, sec. 5-6], and the integration is taken over the cross section area S^l of P^l ($S^l = C^l$ and $ds' = dl'$ for any P^l of zero thickness). Inside each S^m, however, the total electric field, incident plus scattered, must be equal to E_y^m:

$$E_y^i + E_y^s = E_y^m, \qquad (x,z)\in S^m,\ m = 1,2,\cdots,p. \tag{7}$$

Consequently

$$\sin\left(\frac{\pi}{a}x\right)e^{-\gamma_1 z} = -j\omega\mu\left(\phi^m(x,z) - \left(\frac{\omega}{c_0}\right)^2\cdot\sum_{l=1}^{P}(\epsilon_r^l - \epsilon_r)\int_{S^l}\phi^l(x',z')G(x,z|x',z')\,ds'\right),$$
$$(x,z)\in S^m,\ m = 1,2,\cdots,p \tag{8}$$

which is the required integral equation for ϕ^1 through ϕ^P.

The higher order ($n>1$) modes excited are evanescent, i.e., decay exponentially with distance from the posts. Thus, at sufficiently large distances, only the dominant ($n=1$) mode can exist in the waveguide. The reflection coefficient of the dominant mode is readily found from (1), (5), and (6) as

$$\Gamma=-\frac{j\omega\mu}{a\gamma_1}\left(\frac{\omega}{c_0}\right)^2\sum_{l=1}^{p}\left(\epsilon_r^l-\epsilon_r\right)\cdot\int_{S^l}\phi^l(x',z')\sin\left(\frac{\pi}{a}x'\right)e^{-\gamma_1 z'}\,ds'. \quad (9)$$

The transmission coefficient of the dominant mode is then

$$T=1-\frac{j\omega\mu}{a\gamma_1}\left(\frac{\omega}{c_0}\right)^2\sum_{l=1}^{p}\left(\epsilon_r^l-\epsilon_r\right)\cdot\int_{S^l}\phi^l(x',z')\sin\left(\frac{\pi}{a}x'\right)e^{\gamma_1 z'}\,ds'. \quad (10)$$

III. The Scattering Matrix

Following Montgomery *et al.* [15, sec. 5-14], the scattering matrix of the posts is defined as

$$S=\begin{bmatrix} S_{11} & S_{12}\\ S_{21} & S_{22}\end{bmatrix}. \quad (11)$$

In (11), S_{11} and S_{21} are, respectively, the amplitudes of the dominant waveguide mode reflected to the left and transmitted to the right of the posts. Thus, S_{11} and S_{21} are given by (9) and (10), respectively.

Similarly, S_{22} and S_{12} are, respectively, the reflection and transmission coefficients of a dominant waveguide mode incident on the posts from the right. This mode has only a y-component of electric field, which is given by

$$E_y^i=\sin\left(\frac{\pi}{a}x\right)e^{\gamma_1 z}. \quad (12)$$

The previous analysis carried through in this case. Thus, the electric field scattered in the waveguide is given by (5) where ϕ^1 through ϕ^p are now determined by solving the integral equation

$$\sin\left(\frac{\pi}{a}x\right)e^{\gamma_1 z}=-j\omega\mu\left(\phi^m(x,z)-\left(\frac{\omega}{c_0}\right)^2\sum_{l=1}^{p}\left(\epsilon_r^l-\epsilon_r\right)\cdot\int_{S^l}\phi^l(x',z')G(x,z|x',z')\,ds'\right),$$
$$(x,z)\in S^m,\ m=1,2,\cdots,p. \quad (13)$$

The scattering parameters S_{22} and S_{12} are then given by

$$S_{22}=-\frac{j\omega\mu}{a\gamma_1}\left(\frac{\omega}{c_0}\right)^2\sum_{l=1}^{p}\left(\epsilon_r^l-\epsilon_r\right)\cdot\int_{S^l}\phi^l(x',z')\sin\left(\frac{\pi}{a}x'\right)e^{\gamma_1 z'}\,ds' \quad (14)$$

$$S_{12}=1-\frac{j\omega\mu}{a\gamma_1}\left(\frac{\omega}{c_0}\right)^2\sum_{l=1}^{p}\left(\epsilon_r^l-\epsilon_r\right)\cdot\int_{S^l}\phi^l(x',z')\sin\left(\frac{\pi}{a}x'\right)e^{-\gamma_1 z'}\,ds'. \quad (15)$$

IV. The Equivalent Network

Let dominant waveguide modes of arbitrary amplitudes c_1 and c_2 be incident on the posts from the left and from the right, respectively. Far from the posts, only the same mode can exist. The z-transverse field in the waveguide at sufficiently large distances can then be written as [12, sec. 8-1]

$$\left.\begin{aligned} \boldsymbol{E}&=\begin{cases} V_1(z)\boldsymbol{e}_1 & z\ll 0\\ V_2(z)\boldsymbol{e}_1 & z\gg 0\end{cases}\\ \boldsymbol{H}&=\begin{cases} I_1(z)\boldsymbol{a}_z\times\boldsymbol{e}_1 & z\ll 0\\ I_2(z)\boldsymbol{a}_z\times\boldsymbol{e}_1 & z\gg 0\end{cases}\end{aligned}\right\} \quad (16)$$

In (16)

$$\left.\begin{aligned} V_{1,2}(z)&=V_{1,2}^{+}e^{-\gamma_1 z}+V_{1,2}^{-}e^{\gamma_1 z}\\ I_{1,2}(z)&=I_{1,2}^{+}e^{-\gamma_1 z}-I_{1,2}^{-}e^{\gamma_1 z}\end{aligned}\right\} \quad (17)$$

where

$$\left.\begin{aligned} V_1^{+}&=\eta_1 I_1^{+}=c_1\\ V_1^{-}&=-\eta_1 I_1^{-}=c_1S_{11}+c_2S_{12}\\ V_2^{+}&=\eta_1 I_2^{+}=c_1S_{21}+c_2S_{22}\\ V_2^{-}&=-\eta_1 I_2^{-}=c_2\end{aligned}\right\} \quad (18)$$

are the mode voltages and currents, and

$$\left.\begin{aligned} \boldsymbol{e}_1&=\sin\left(\frac{\pi}{a}x\right)\boldsymbol{a}_y\\ \eta_1&=\frac{j\omega\mu}{\gamma_1}\end{aligned}\right\} \quad (19)$$

are the mode vector and characteristic impedance of the dominant waveguide mode, respectively.

Let (V_1, I_1) and (V_2, I_2) be, respectively, the amplitudes of $(E_y, -H_x)$ far to the left and to the right of the posts extrapolated back to the $z=0$ plane. It then follows from (16) through (18) that (V_1, V_2) and (I_1, I_2) are related to (c_1, c_2) through the matrix equations

$$\left.\begin{aligned} \vec{V}_d&=(U+S)\vec{c}\\ \eta_1\vec{I}_d&=(U-S)\vec{c}\end{aligned}\right\} \quad (20)$$

where

$$\vec{c}=\begin{bmatrix} c_1\\ c_2\end{bmatrix}\qquad \vec{V}_d=\begin{bmatrix} V_1\\ V_2\end{bmatrix}\qquad \vec{I}_d=\begin{bmatrix} I_1\\ -I_2\end{bmatrix} \quad (21)$$

and U is the identity matrix. The impedance matrix of the posts Z is then defined by [15, sec. 5-9]

$$Z\vec{I}_d=\vec{V}_d. \quad (22)$$

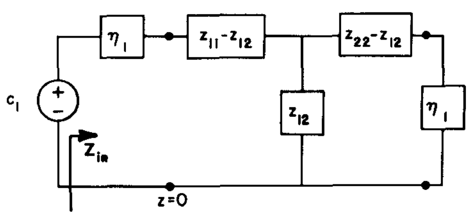


Fig. 2. The equivalent network of the posts.

Consequently

$$Z=\eta_1(U+S)(U-S)^{-1}=\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \tag{23}$$

since $\vec{c}$ is completely arbitrary.

As can be seen, the effect of the posts on the dominant waveguide mode at far points can be described by two transmission lines of characteristic impedance η_1. The voltage and current waves on the transmission lines are given by (17) and (18) and are produced by voltage generators with voltages c_1 and c_2 matched to the waveguide. At $z=0$, however,

$$\left.\begin{aligned} V_{1,2} &= V_{1,2}(0) = V_{1,2}^{+} + V_{1,2}^{-} \\ I_{1,2} &= I_{1,2}(0) = I_{1,2}^{+} - I_{1,2}^{-} \end{aligned}\right\} \tag{24}$$

are related by the impedance matrix Z, a fact that manifests itself in the presence of a two-port T-network connecting the two lines there. For an incident dominant mode from the left only, the equivalent network reduces to that shown in Fig. 2.

V. Solution of the Integral Equation

The integral equation (8) can be written in the operator form

$$V=\phi^m+\sum_{l=1}^{p} D(\phi^l), \qquad m=1,2,\cdots,p \tag{25}$$

where

$$\left.\begin{aligned} V &= -\frac{1}{j\omega\mu}\sin\left(\frac{\pi}{a}x\right)e^{-\gamma_1 z} \\ D(\phi^l) &= -\left(\frac{\omega}{c_0}\right)^2\left(\epsilon_r^l-\epsilon_r\right) \\ &\quad\cdot\int_{S^l}\phi^l(x',z')G(x,z|x',z')\,ds' \end{aligned}\right\}(x,z)\in S^m. \tag{26}$$

An exact solution of (25) can be rarely obtained, and an approximate solution has then to be sought.

Let each C^m be approximated by a polygon Σ^m, and put

$$\bar{S}^m=\bigcup_{k=1}^{q^m}\Delta_k^m \tag{27}$$

where $\bar{S}^m$ is the polygonal cross section of P^m defined by Σ^m, and Δ_k^m denote simplexes (triangles in a two-dimensional space and line segments in a one-dimensional space), as shown in Fig. 3. Put

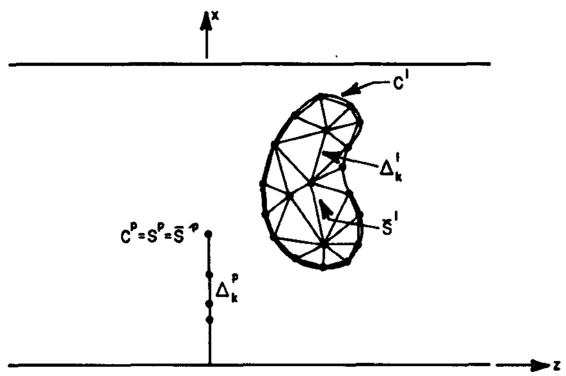


Fig. 3. Modeling of the posts by simplexes.

$$\phi^m\simeq\sum_{k=1}^{q^m}\alpha_k^m\phi_k^m. \tag{28}$$

In (28), each ϕ_k^m is a function, yet to be specified, that is defined on Δ_k^m and vanishes on $\Delta_{j\neq k}^{l\neq m}$, and α_k^m are coefficients to be determined.

Substituting (28) into (25), there results

$$V=\alpha_k^m\phi_k^m+\sum_{l=1}^{p}\sum_{j=1}^{q^l}\alpha_j^l D\left(\phi_j^l\right)+r,$$

$$(x,z)\in\Delta_k^m,\quad m=1,2,\cdots,p,\quad k=1,2,\cdots,q^m. \tag{29}$$

The integration in (29) is taken over Δ_j^l rather than S^l, and r is a residual term. A point-matching solution [16, sec. 1-4] is obtained by requiring that

$$r(x_k^m,z_k^m)=0 \tag{30}$$

for some $(x_k^m,z_k^m)\in\Delta_k^m$. Taking (30) into consideration, (29) then becomes

$$\overline{\overline{Z}}\vec{I}=\vec{V}. \tag{31}$$

In (31), $\vec{V}$ and $\vec{I}$ are p-segment vectors whose mth and lth segments are, respectively, the $q^m\times 1$ and $q^l\times 1$ vectors

$$\vec{V}^m=\left[V_k^m\right]=\left[V(x_k^m,z_k^m)\right] \tag{32}$$

$$\vec{I}^l=\left[\alpha_j^l\right] \tag{33}$$

and $\overline{\overline{Z}}$ is a $p\times p$ block matrix whose mlth block is the $q^m\times q^l$ matrix

$$\overline{\overline{Z}}^{ml}=\left[Z_{kj}^{ml}\right]=\left[\left(\phi_j^l+D\left(\phi_j^l\right)\right)(x_k^m,z_k^m)\right]. \tag{34}$$

A point-matching solution of (13) can be obtained in a similar manner. Clearly, using the same functions ϕ_k^m for expansion, there results a matrix equation for the coefficients of expansion similar to (31), but with $-\vec{V}^*$ replacing $\vec{V}$, where "$*$" denotes complex conjugate.

VI. Evaluation of Matrix Elements

The evaluation of matrix elements constitutes the major portion of the work involved in the numerical solution. An efficient evaluation of the elements of the moment matrix is therefore necessary for the success of the solution.

A typical element in the moment matrix is given by

$$Z_{kj}^{ml} = \phi_j^l(x_k^m, z_k^m) + W^l \int_{\Delta_j^l} \phi_j^l(x', z') G(x_k^m, z_k^m | x', z')\, ds' \tag{35}$$

where W^l is a constant that can be identified from (26), and ϕ_j^l are yet to be specified. A particularly simple choice for ϕ_j^l is

$$\phi_j^l(x,z) = \begin{cases} 1 & \text{if } (x,z) \in \Delta_j^l \\ 0 & \text{otherwise} \end{cases} \tag{36}$$

which corresponds to a pulse expansion of the polarization currents. Furthermore, the matching points are taken to be the centroids of the triangles and/or the midpoints of the line segments. Thus,

$$Z_{kj}^{ml} = \delta_{ml}\delta_{kj} + W^l \int_{\Delta_j^l} G(x_k^m, z_k^m | x', z')\, ds' \tag{37}$$

where δ_{kj} is the Kronecker delta function (one if $k = j$ and zero if $k \neq j$).

The evaluation of the matrix element proceeds by writing G in the form [1]

$$G = G_d + G_s + G_c. \tag{38}$$

In (38)

$$\left.\begin{aligned}
G_d(x_k^m, z_k^m | x', z') &= \frac{1}{j\pi\beta_1} \sin\left(\frac{\pi}{a} x_k^m\right) \sin\left(\frac{\pi}{a} x'\right) e^{-j\pi/a |z_k^m - z'|\beta_1} \\
G_s(x_k^m, z_k^m | x', z') &= \frac{1}{4\pi} \log\left(\frac{\cosh\left(\frac{\pi}{a}(z_k^m - z')\right) - \cos\left(\frac{\pi}{a}(x_k^m + x')\right)}{\cosh\left(\frac{\pi}{a}(z_k^m - z')\right) - \cos\left(\frac{\pi}{a}(x_k^m - x')\right)} \right) \\
G_c(x_k^m, z_k^m | x', z') &= -\frac{1}{\pi} \sin\left(\frac{\pi}{a} x_k^m\right) \sin\left(\frac{\pi}{a} x'\right) e^{-\pi/a|z_k^m - z'|} \\
&\quad - \frac{1}{\pi} \sum_{n=2}^{\infty} \sin\left(n\frac{\pi}{a} x_k^m\right) \sin\left(n\frac{\pi}{a} x'\right) \left(\frac{e^{-\pi/a|z_k^m - z'|\beta_n}}{\beta_n} - \frac{e^{-n\pi/a|z_k^m - z'|}}{n} \right)
\end{aligned}\right\} \tag{39}$$

where

$$\left.\begin{aligned}
\gamma_1 &= j\sqrt{\kappa^2 - \left(\frac{\pi}{a}\right)^2} = j\frac{\pi}{a}\sqrt{\left(\frac{2a}{\lambda}\right)^2 - 1} = j\frac{\pi}{a}\beta_1 \\
\gamma_n &= \sqrt{\left(n\frac{\pi}{a}\right)^2 - \kappa^2} = \frac{\pi}{a}\sqrt{n^2 - \left(\frac{2a}{\lambda}\right)^2} = \frac{\pi}{a}\beta_n, \qquad n \geq 2
\end{aligned}\right\} \tag{40}$$

and "log" denotes natural logarithm. The decomposition (38) amounts to expressing the dynamic Green's function G in terms of a dominant mode wave G_d, the corresponding static Green's function G_s, plus a correction series G_c. The correction series is dominated by an exponentially convergent series of positive monotonically decreasing terms and can therefore be directly summed at a minimal cost [1].

The evaluation of the matrix element is completed by integrating the Green's function, as given by (38), numerically, where appropriate quadrature rules can be used. When evaluating any diagonal element, however, the static component of G_s offers a logarithmic singularity that requires particular attention. Put

$$G_s = G_{ss} + (G_s - G_{ss}) = G_{ss} + G_{sp} \tag{41}$$

where

$$G_{ss}(x_k^m, z_k^m | x', z') = -\frac{1}{2\pi} \log\left(\frac{\pi}{a} \sqrt{(x_k^m - x')^2 + (z_k^m - z')^2} \right) \tag{42}$$

is the singular part of G_s. G_{ss} is readily integrated over a simplex to give [17]

$$\int_{\Delta_k^m} G_{ss}(x_k^m, z_k^m | x', z')\, ds' = -\frac{1}{2\pi} A_k^m \log\left(\frac{\pi}{a}\right) - \frac{1}{4\pi} \sum_{i=1}^{3} \sum_{j=1}^{2} \left((-)^j f_i^0 \left(l_i^j \log(f_i^j) + \tan^{-1}\left(\frac{l_i^j}{f_i^0}\right) \right) - 1.5 l_i^j \right) \tag{43}$$

if Δ_k^m is a triangle, and

$$\int_{\Delta_k^m} G_{ss}(x_k^m, z_k^m | x', z')\, ds' = -\frac{1}{2\pi} L_k^m \left(\log\left(\frac{\pi}{2a} L_k^m\right) - 1 \right) \tag{44}$$

if Δ_k^m is a line segment of length L_k^m. The quantities in (43) are defined in Fig. 4. G_{sp} has no singularity and can therefore be integrated over Δ_k^m using quadratures.

VII. Numerical Results

A user-oriented computer code has been written and applied to a large set of test problems with a wide variety of post configurations. Only a few of the results obtained for circular posts free from losses and located in a rectan-

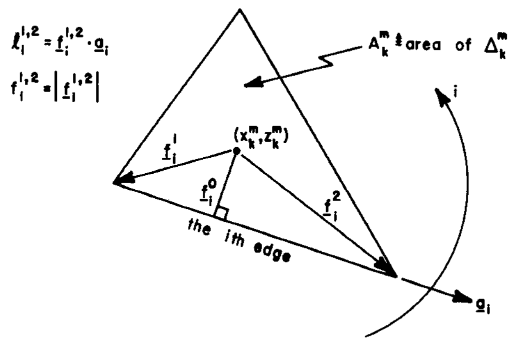


Fig. 4. Definition of the quantities in (43).

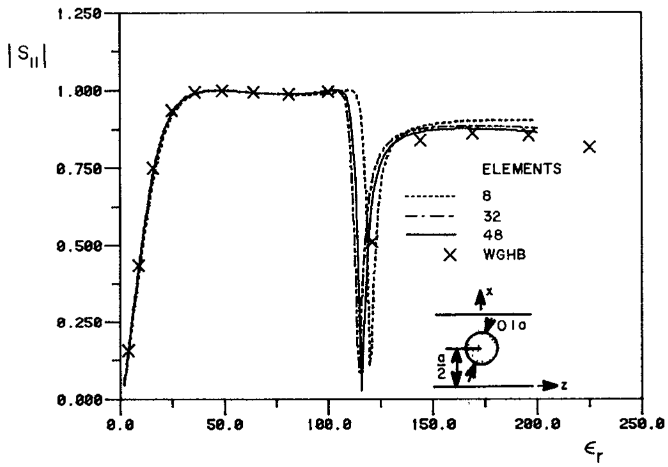


Fig. 5. The convergence of the moment procedure for a centered circular post of diameter $d = 0.10a$, $\lambda = 1.4a$.

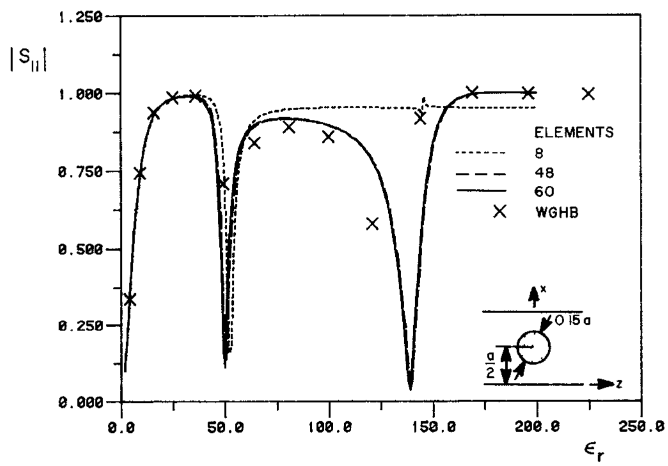


Fig. 6. The convergence of the moment procedure for a centered circular post of diameter $d = 0.15a$, $\lambda = 1.4a$.

gular waveguide whose medium is the vacuum ($\epsilon = \epsilon_0$) are given in this section. More results can be found in [18].

The actual computations follow the evaluation steps in Section VI. The numerical integration is carried out using third-order closed Silvester quadratures for integration over simplexes [19]. In a one-dimensional space, they become the familiar Newton–Cotes quadratures. One of the nodes for this particular order, however, is located at the centroid of the simplex, which necessitates a careful evaluation of

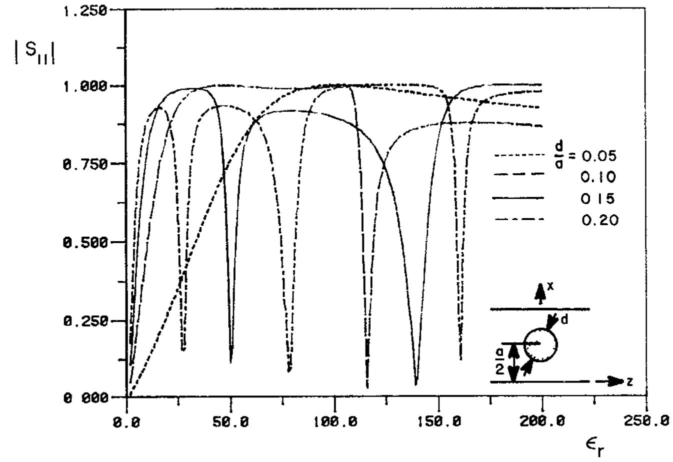


Fig. 7. The change of $|S_{11}|$ for the centered circular post with ϵ_r, $\lambda = 1.4a$.

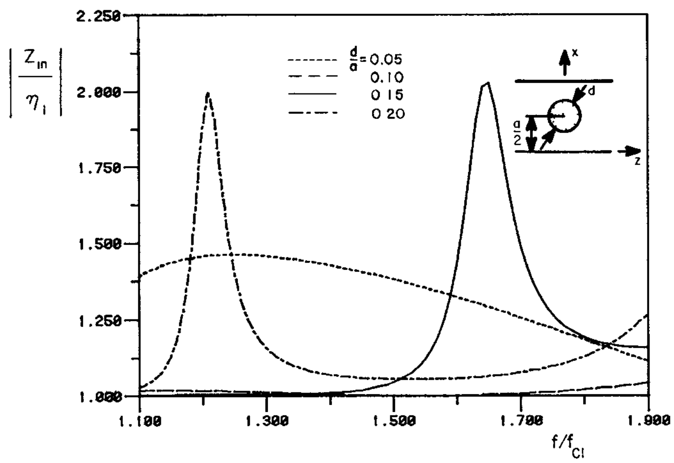


Fig. 8. The change of $|Z_{in}|$ for the centered circular post with frequency, $\epsilon_r = 38.0$.

G_{sp} at this node. At the centroid of Δ_k^m, G_{sp} is readily evaluated as

$$G_{sp}(x_k^m, z_k^m | x_k^m, z_k^m) = \frac{1}{2\pi} \log\left(2\sin\left(\frac{\pi}{a} x_k^m\right)\right). \quad (45)$$

The scattering parameters and the elements of the equivalent network are basically the quantities to be computed. In the course of computation, a large array of testing procedures are conducted. Because of the approximations involved in the numerical solution, the scattering matrix need no longer be symmetric, nor is it necessarily unitary for posts that are free from losses. The unitary condition has been found satisfied to within an error of magnitude $O(10^{-14})$ for a double precision arithmetic mode of operation, while $|S_{21} - S_{12}|$ has always been $O(10^{-7})$. Furthermore, the results obtained compare very well with the data in the *Waveguide Handbook* (WGHB) [6] where they are accurate, i.e., away from any resonances, as can be readily seen from Figs. 5 and 6. It is interesting to note that the solution converges rather quickly up to the first resonance with only a few triangular elements needed. This has been observed to be true for circular posts of diameter $d \leqslant 0.15a$.

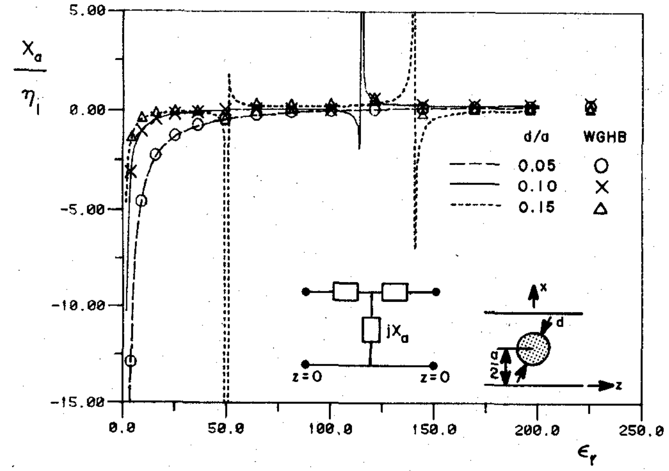
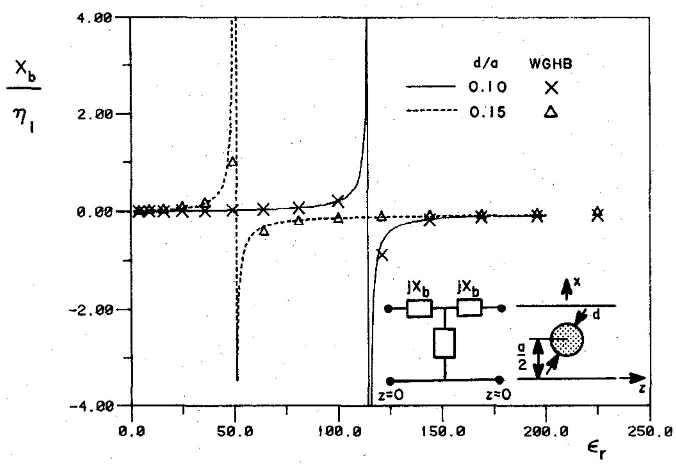
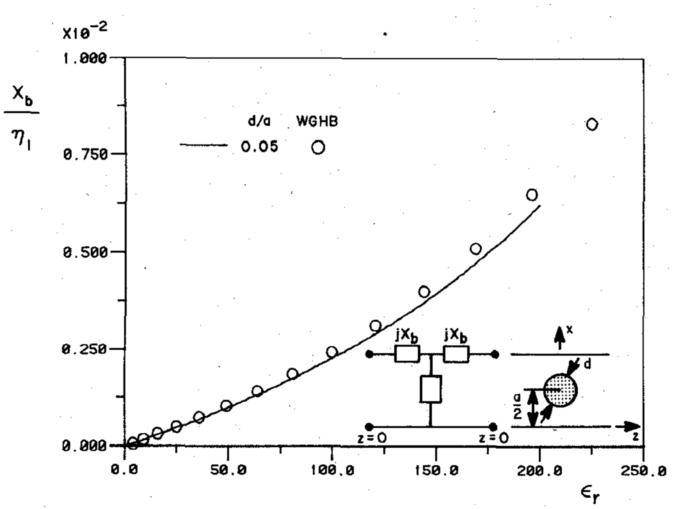


Fig. 9. The change of the parallel reactance for the centered circular post with ϵ_r, $\lambda = 1.4a$.



(a)



(b)

Fig. 10. The change of the series reactance for the circular centered post with ϵ_r, $\lambda = 1.4a$.

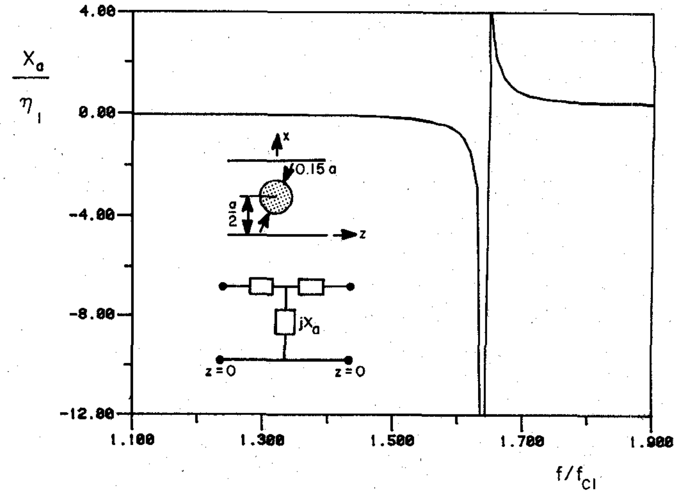


Fig. 11. The change of the parallel reactance for a centered circular post of diameter $d = 0.15a$ with frequency, $\epsilon_r = 38.0$.

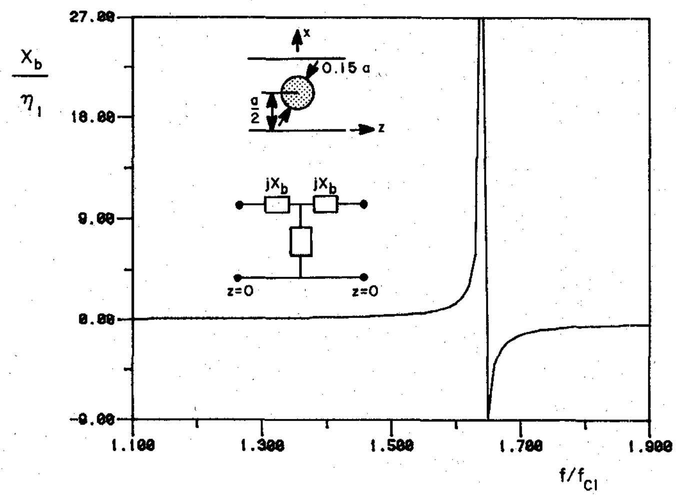


Fig. 12. The change of the series reactance for a centered circular post of diameter $d = 0.15a$ with frequency, $\epsilon_r = 38.0$.

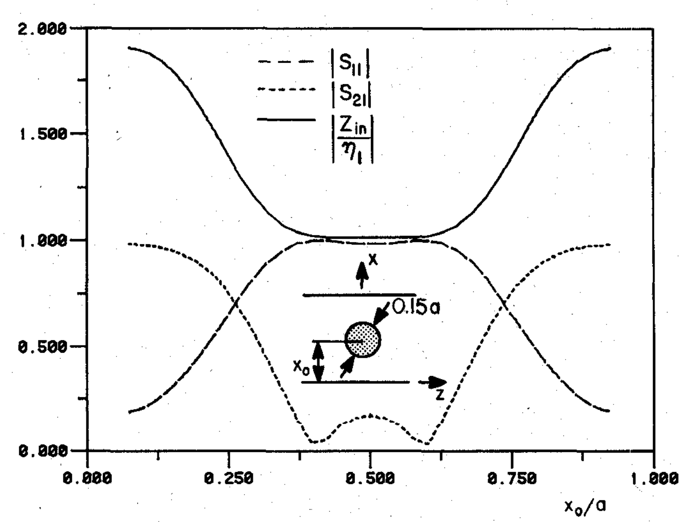


Fig. 13. The change of $|S_{11}|$, $|S_{21}|$, and $|Z_{in}|$ for a centered circular post of diameter $d = 0.15a$ with location, $\epsilon_r = 38.0$, $\lambda = 1.4a$.

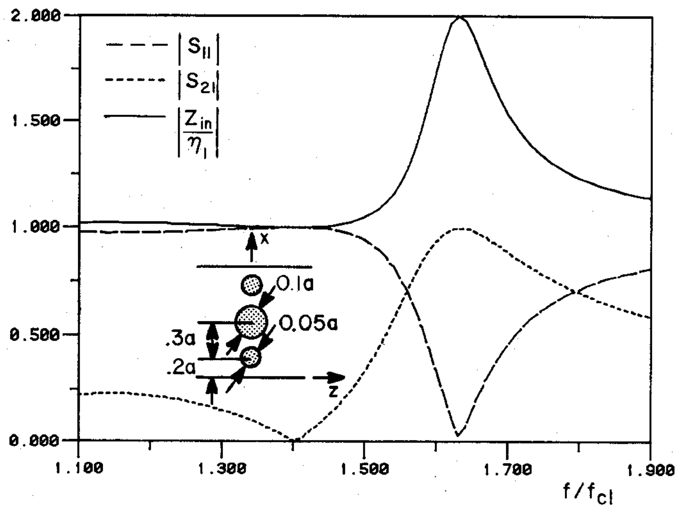


Fig. 14. The change of $|S_{11}|$, $|S_{21}|$, and $|Z_{in}|$ for a symmetric triple-post configuration with frequency, $\epsilon_r = 38.0$.

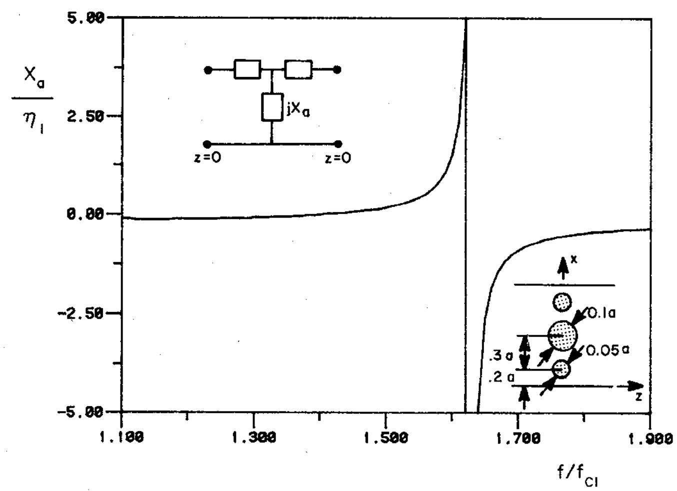


Fig. 15. The change of the parallel reactance for a symmetric triple-post configuration with frequency, $\epsilon_r = 38.0$.

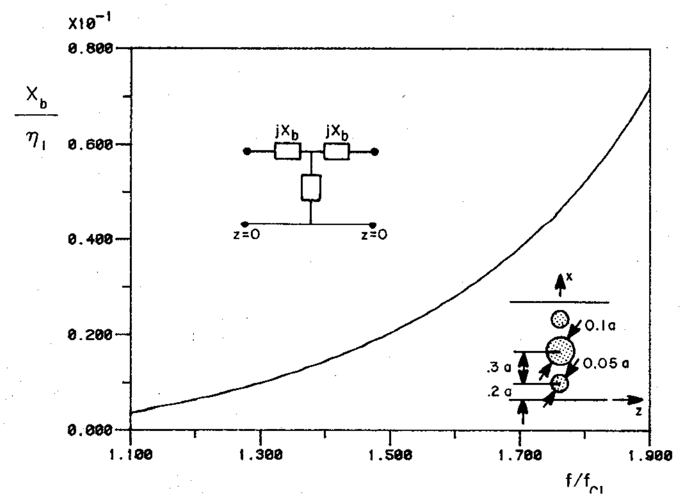


Fig. 16. The change of the series reactance for a symmetric triple-post configuration with frequency, $\epsilon_r = 38.0$.

A reasonably accurate evaluation of the first resonance for such posts is therefore possible with only about eight elements. More elements, however, are needed to accurately determine higher-order resonances.

Fig. 7 displays the change of $|S_{11}|$ with ϵ_r for centered posts of diameter $d = 0.05a$, $0.1a$, $0.15a$, and $0.20a$. The change of $|Z_{in}|$ with frequency for the same posts is shown in Fig. 8. The branch reactances for centered posts of diameter $d = 0.05a$, $0.1a$, and $0.15a$ are plotted versus ϵ_r in Figs. 9 and 10, and versus frequency in Figs. 11 and 12 for a centered post of diameter $d = 0.15a$. The change of $|S_{11}|$, $|S_{21}|$, and $|Z_{in}|$ for a post of diameter $d = 0.15a$ with location is shown in Fig. 13. Finally, the change of the scattering parameters and network reactances with frequency for a symmetric triple-post configuration with varied diameters is shown in Figs. 14, 15, and 16. In these figures, ϵ_r changes between 2.0 and 200.0 with increments of 1.0 for $\lambda = 1.4a$, and frequency changes between $1.1f_{c1}$ and $1.9f_{c1}$ with increments of $0.01f_{c1}$ for $\epsilon_r = 38.0$, where f_{c1} is the cutoff frequency of the dominant waveguide mode. Thus, a total of 200 points is included in each curve in Figs. 5–7, 9, and 10, and a total of 81 points is included in Figs. 8, 11, 12, and 14–16. Furthermore, a total of 181 points is included in each curve in Fig. 13, the majority of which are taken close to the waveguide walls.

VIII. Resonance

The dielectric posts are very resonant structures. These resonances are conveniently characterized by the quality factor of the posts. The quality factor is defined as

$$Q = \omega \frac{\text{total energy stored inside and outside the posts}}{\text{total power lost}}. \tag{46}$$

In (46), the total power lost is the sum of that due to heating caused by the nonvanishing conductivity of the posts and the power carried by the scattered dominant waveguide mode. The quality factor is therefore a positive continuous function of all the parameters involved in (46). The values of any parameter for which Q is maximum are called resonances. This definition differs from that advanced by Richtmyer [20], which considers only the energy stored inside the posts. The difference is actually in the interpretation of the meaning of stored energy. The definition as given in (46) allows using the matrix representations of the posts since they incorporate the energy stored inside the posts as well as that stored in the evanescent higher-order modes outside the posts. The resonances can then be readily computed from the $|Z_{in}|$ curves in a manner similar to that which is normally followed in network theory. As an example, the resonant frequency for a centered dielectric post of diameter $d = 0.15a$ and dielectric constant $\epsilon_r = 38.0$ is readily found from Fig. 8 to be $1.65f_{c1}$. Other parameters can also be determined for this post. For instance, a bandwidth of $0.11f_{c1}$ and a loaded quality factor of 15.31 are readily computed. The corresponding numbers for a centered post of diameter $d =$

$0.20a$ and the same dielectric constant are $1.21f_{c1}$, $0.07f_{c1}$, and 17.44, respectively.

Other types of resonance may also occur, namely, branch resonances. Such resonances can be determined from the data obtained as well. As a matter of fact, branch resonances occur whenever post resonances exist, and at the same frequency or permittivity, or in the close vicinity of it. For instance, the series branch for a centered circular post of diameter $d = 0.15a$ is antiresonant at $f = 1.65f_{c1}$ and $\epsilon_r = 38.0$. An examination of Fig. 8 against Fig. 12 further confirms this assertion. Care, however, should be exercised when identifying the branch resonances. The reactance X_a of the same post shown in Fig. 11 has no resonances since it does not obey Foster's theorem where the irregularity occurs.

Some of the dielectric posts exhibit frequency filtering characteristics near resonances. The determination of resonances for different configurations therefore allows for the design of new types of microwave filters. Equally important is the investigation of methods for resonating nonresonant structures. One method, readily suggested by the results presented, is to utilize other posts to resonate nonresonant posts. For instance, a centered circular post of diameter $d = 0.10a$ and $\epsilon_r = 38.0$ is not resonant, but can be made so by placing two posts of diameter $d = 0.05a$ and the same dielectric constant symmetrically about it, as can be seen from Figs. 14 and 15. Because of the typically low quality factor for the posts, however, such setups may not be used in filter design as they stand. Methods for Q-enhancement and filter design using dielectric posts are the subject of a coming paper.

IX. Summary

A complete field analysis of a system of dielectric posts in a rectangular waveguide has been given. A user-oriented computer program has been written and applied to a large set of problems with a wide variety of post configurations, thereby providing a large set of design data that have not been available before.

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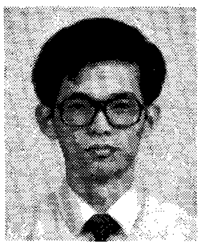

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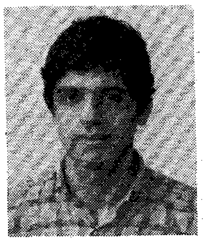

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